ON SOLVING A DYNAMIC OPTIMAL BUDGETING PROBLEM

by

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RESOURCE DYNAMICS
GWU/IMSE/Serial T-474/83
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THE GEORGE WASHINGTON UNIVERSITY
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1. Introduction and Summary

In this paper we wish to establish a solution procedure designed to solve a model which seeks to maximize a measure of asset value of owned resources. These resources are updated over time, and change according to the amount of manpower and/or maintenance supplied.

The specific model which we address is a slight modification of a model formulated by Rolf Clark in a pair of notes ([1], [2]). It is a dynamic model with a system of difference equations linking the variables in adjacent time periods. The objective function (to be maximized) is a discounted sum over time periods of a measure of the "effective asset value" of the owned resources.

Viewed as a mathematical optimization problem, the model appears to be a difficult nonconvex optimization problem. In Section 2, we will establish the notation and the equations describing the model.

The nature of the problem is such that we were able to accurately approximate the objective function and design a branch and bound method to solve the problem. The details of this "decoupling" are presented in Section 3.

A moderately expensive way to implement the algorithm suggested by the results of Section 3 would be to design (or, perhaps, modify) a code specifically for the problem. Fortunately, we were able to effect a solution of a small model by means of a 0-1 code existing at GWU (LINDO). The results are presented in Section 4.

2. The Model

The model presented here is based on Clark's notes [1] and [2], and also on a Technical Memorandum by one of the author [3].

State Variables

B_t	budget at time t
P_t	procurement at time t
MXS_t	maintenance supplied at time t
$MBLOG_t$	maintenance backlog at time t
MX_t	maintenance demand during year t
MS_t	manpower supplied during year t
M_t	manpower demand during year t
AV_t	asset value at time t
R_t	assets retired at time t
EAV_t	effective asset value at time t

The state variables are determined by a system of difference equations, initial values, and settings of the control variables.

Control Variables

$F1_t$	fraction of maintenance supplied/maintenance demanded
$F2_t$	fraction of manpower supplied/manpower demanded

The model to be presented has a number of parameters which are set exogenously. These parameters are listed below, with values set for the illustrative example of Section 4.

Parameters

$$\left.\begin{array}{lcl} c & = & 1000 \\ \alpha_1 & = & 0.03 \\ \alpha_2 & = & 2 \\ \alpha_3 & = & 29/30 \\ \alpha_4 & = & 0.04 \\ \alpha_5 & = & -0.01 \\ \alpha_6 & = & 25\text{x}10^{-6} \\ \alpha_7 & = & 0.5 \end{array}\right\} \text{current values}$$

$T = 5$ (number of time periods)

Initial Values

$B_0 = 10$

$AV_0 = 100$

$MBLOG_0 = 1$

The relations which link the variables over the time periods are now presented. The basic assumptions underlying and defining the model are exhibited in these relations, and are briefly discussed after the relations are displayed. The reader might note that a number of these relations do <u>not</u> represent constraints on the problem but are definitional. Indeed, the first order of business will be to simplify the relations.

Relations

The maintenance and manpower relations are

$$(1)\quad B_t = B_0(1 + \alpha_1)^t$$

$$(2)\quad P_t = B_t - MXS_t - MS_t$$

$$(3)\quad MXS_t = F1_t\ (MBLOG_{t-1} + MX_t)$$

$$(4)\quad MBLOG_t = \alpha_3^{\alpha_2}\ MBLOG_{t-1} + MX_t - MXS_t$$

$$(5)\quad MX_t = \alpha_4(AV_{t-1} - \alpha_7\ MBLOG_{t-1})$$

$$(6)\quad MS_t = F2_t\ M_t$$

$$(7)\quad M_t = c(1 + \alpha_5)^t\ (AV_{t-1})\ \alpha_6$$

$$(8)\quad AV_t = AV_{t-1} + P_t - R_t$$

$$(9)\quad R_t = AV_{t-1}\ (1-\alpha_3)$$

Relation (1) implies that the budget at time t is a result of an initial budget growing at a yearly rate of α_1. Relation (2) defines procurement at time t to be the difference between the budget at t and the amounts allocated to manpower and maintenance during time period t.

Relations (3) and (6) are best interpreted as defining the ratios $F1_t$ and $F2_t$ as the fraction of maintenance and manpower supplied over the amounts of these quantities demanded.

The maintenance backlog at time t defined by (4) is interpreted as a fraction of the backlog in the previous time period plus the maintenance demand during time t minus the maintenance supplied during this time period. The maintenance demand during period t is interpreted by (5) to be a fixed fraction α_4 of asset value minus a fraction $\alpha_4 \cdot \alpha_7$ of the maintenance backlog of the previous time period.

The manpower demand as defined in (7) is a multiple of the asset value of the previous time period, and the asset value in (8) is updated from the asset value of the previous time period by adding total procurements and subtracting assets retired.

Finally, assets retired in time period t are assumed to be some fixed fraction of the asset value of the previous time period.

The reader should note that an extension of this model which allowed for time-dependent parameters could easily be incorporated, if meaningful parameters could be estimated.

We are now in a position to simplify the relations, by eliminating variables which are dependent and not constrained.

First, we may let MXS_t and MS_t denote the control variables (instead of $F1_t$ and $F2_t$) . We note that the above system can be simplified considerably if we let $MBLOG_t$ and AV_t be state variables. For we may rewrite (4) as

$$\begin{aligned} MBLOG_t &= \alpha_3^{\alpha_2} \ MBLOG_{t-1} + \alpha_4(AV_{t-1} - \alpha_7 \ MBLOG_{t-1}) \\ &\qquad - MXS_t \qquad \text{by (5)} \\ &= (\alpha_3^{\alpha_2} - \alpha_4\alpha_7) \ MBLOG_{t-1} + \alpha_4 \ AV_{t-1} - MXS_t \end{aligned}$$

and we can rewrite (8) as

$$\begin{aligned} AV_t &= AV_{t-1} + (B_t - MXS_t - MS_t) - (1-\alpha_3)AV_{t-1} \quad \text{by (2) and (9)} \\ &= \alpha_3 AV_{t-1} + B_0(1+\alpha_1)^t - MXS_t - MS_t \qquad \text{by (1)} \end{aligned}$$

In order to simplify notation, we set

$$\begin{aligned} MXS_t &= x_t \\ MS_t &= s_t \\ AV_t &= a_t \\ MBLOG_t &= m_t \end{aligned}$$

and get

$$(10) \quad a_t = \alpha_3 a_{t-1} \quad - x_t - s_t + B_0(1+\alpha_1)^t$$

$$(11) \quad m_t = \alpha_4 a_{t-1} + (\alpha_3^{\alpha_2} - \alpha_4\alpha_7)m_{t-1} - x_t$$

$$(12) \quad 0 \leqq x_t \leqq \alpha_4 a_{t-1} + (1-\alpha_4\alpha_7)m_{t-1}$$

$$(13) \quad 0 \leqq s_t \leqq c(1+\alpha_5)^t \, \alpha_6 a_{t-1}$$

where (12) and (13) come from the restrictions $F1_t, F2_t \ \varepsilon \ [0,1]$ together with the defining relations (3), (5), (6), and (7). We can simplify notation further, by re-defining the coefficients above so that

$$(14) \quad a_t = k_1 a_{t-1} \quad - x_t - s_t + B_0(1+\alpha_1)^t$$

$$(15) \quad m_t = k_2 a_{t-1} + k_3 m_{t-1} - x_t$$

$$(16) \quad 0 \leqq x_t \leqq k_4 a_{t-1} + k_5 m_{t-1}$$

$$(17) \quad 0 \leqq s_t \leqq k_{6t} a_t{-1}$$

Note that (14) and (15) define the state variables $a_1,\ldots,a_t$ and $m_1,\ldots,m_t$ as <u>linear</u> functions of the control variables. Also (16) and (17) represent a system of <u>state constrained</u> upper bounds on the control variables.

Equations (14) and (15), inequalities (16) and (17), and initial conditions $a_0 = AV_0$, $m_0 = MBLOG_0$ constitute a set of <u>linear</u> constraints. We are now in a position to address the objective function.

Empirical evidence has suggested that a meaningful objective might have the form

$$(18) \quad \sum_{t=1}^{T} w_t \; EAV_t$$

where $w_1,\ldots,w_T$ are preassigned "weights" and EAV_t is some measure of the "effective asset value" at time t. It has been suggested (Clark [2]) that EAV_t should be the asset value AV_t at time t, diminished by some function of (a) the ratio of the maintenance backlog over the asset value, and (b) the ratio of the manpower supplied over the asset value. We assume that

$$EAV_t = AV_t \; \min \{f(MBLOG_t/AV_t) \; , \; g(MS_t/M_t)\}$$

or, in the simplified notation,

$$EAV_t = a_t \min \{f(m_t/a_t) \; , \; g(s_t/\ell_t a_{t-1})\}$$

where f and g are to be described, and $\ell_t = c(1+\alpha_5)^t \alpha_6$ (see equation (7)).

At this point, we wish to emphasize that the exact form of f and g is immaterial to the algorithm which follows. The algorithm applies to <u>any</u> given function f and g.

A function f which has a form compatible with the current problem ($f(0) = 1$ when there is no backlog, $f(1) = 0$ when the asset value equals the backlog) is

$$(19) \quad f(v) = (1+58v)(1-v)^{58}$$

and the function g which we use here is

$$(20) \quad g(r) = r^5(6-5r)$$

(following Soland's note [5]). These functions are depicted in Figure 1.

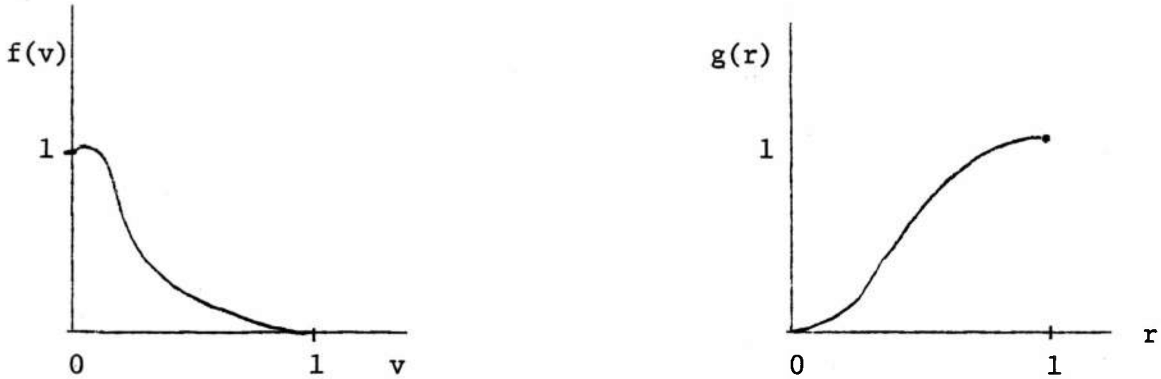

Figure 1. Example Functions f and g

In summary, the model which we address has the form

$$\underset{x_t, s_t \geq 0}{\text{maximize}} \sum_{t=1}^{T} w_t a_t \min \{f(m_t/a_t)\ ,\ g(s_t/\ell_t a_{t-1})\}$$

subject to constraint (14), (15), (16), (17) .

Note that this problem is equivalent to the <u>problem P</u>:

$$\underset{x_t, s_t, t_t, v}{\text{maximize}} \quad v$$

$$\text{subject to} \quad v \leq \sum_{t=1}^{T} t_t$$

$$\left.\begin{array}{ll} (21) & t_t \leq w_t\ a_t\ f(m_t/a_t) \\ (22) & t_t \leq w_t\ a_t\ g(s_t/\ell_t a_{t-1}) \end{array}\right\} \quad t=1,\ldots,T$$

and constraints (14), (15), (16) and (17).

We note that this is a nonconvex optimization problem as the functions on the right-hand side of the above inequalities are generally <u>not</u> concave.

3. Solving the Model

In this section, we define an approximation to problem P which can be solved globally, and indicate a method that could be used to solve the problem.

We first address the function $f(v)$. Let $0 = \bar{v}_0 < \bar{v}_1 < \bar{v}_2 < \ldots < \bar{v}_n = 1$ be a set of $n+1$ points partitioning the interval $[0,1]$ into n subintervals. We wish to define the piecewise linear approximation to $f(v)$ which agrees with f at $\bar{v}_0,\ldots,\bar{v}_n$, and linearly interpolates adjacent values. Figure 2 exhibits the approximation desired.

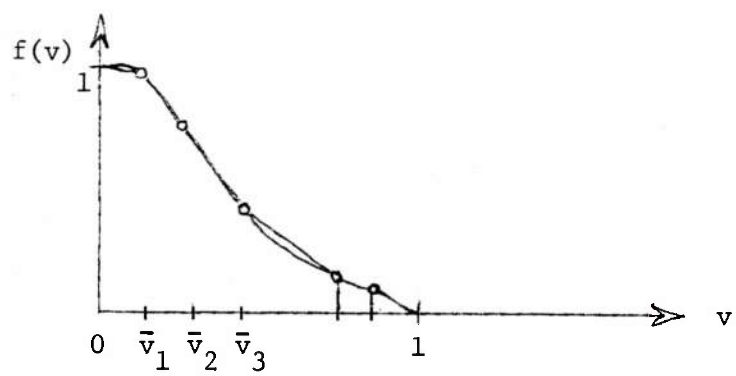

Figure 2. Approximation of f

We introduce variables $\theta_{t0}, \theta_{t1},\ldots,\theta_{tn}$ associated with the points $\bar{v}_0,\bar{v}_1,\ldots,\bar{v}_n$, and note that

$$f(v_t) \cong \sum_{j=0}^{n} \theta_{tj} f(\bar{v}_j)$$

where

$$(23) \quad v_t = \sum_{j=0}^{n} \theta_{tj} \bar{v}_j$$

$$(24) \quad 1 = \sum_{j=0}^{n} \theta_{tj}$$

$$\theta_{tj} \geqq 0 \qquad (j=0,1,\ldots,n)$$

<u>and</u> at most two θ_{tj}'s can be positive, and if two are positive, they must correspond to <u>adjacent</u> grid points bordering v_t. This latter restriction is known as the <u>adjacent weights restriction</u> (AWR), and is used in [4].

We wish to replace the function f in problem P by its piecewise linear approximation. Thus, the term

$$a_t \, f(m_t/a_t)$$

becomes

$$\sum_{j=0}^{n} [\theta_{tj} a_t] \, f(\bar{v}_j)$$

and the new constraint (23) becomes

$$\frac{m_t}{a_t} = \sum_{j=0}^{n} \theta_{tj} \bar{v}_j$$

i.e.,

$$m_t = \sum_{j=0}^{n} [\theta_{tj} a_t] \, \bar{v}_j$$

Setting

$$(25) \quad z_{tj} = \theta_{tj} a_t$$

constraint (21) becomes

$$(26) \quad t_t \leqq w_t \sum_{j=0}^{n} z_{tj} f(\bar{v}_j)$$

$$(27) \quad m_t = \sum_{j=0}^{n} z_{tj} \bar{v}_j$$

$$(28) \quad a_t = \sum_{j=0}^{n} z_{tj}$$

where the latter constraint results by summing (25) over j , and using (24).

We use similar manipulations to handle constraint (22), but we need to make an additional approximation. We replace a_{t-1} by a_t in (22) in order that the construct parallel to (25) will result in linear equations similar to (26), (27), and (28).

Let $0 = \bar{r}_0 < \bar{r}_1 < \ldots < \bar{r}_m = 1$ be a set of $m+1$ points partitioning the interval [0,1] into m subintervals. Introducing variables ω_{tj} corresponding to these subdivision points for each time period, we have

$$g_t(r_t) \cong \sum_{j=0}^{m} \omega_{tj} \bar{r}_j$$

if

$$r_t = \sum_{j=0}^{m} \omega_{tj} \bar{r}_j$$

$$1 = \sum_{j=0}^{m} \omega_{tj}$$

$$\omega_{tj} \geqq 0$$

and $\omega_{t0}, \ldots, \omega_{tm}$ satisfy the AWRs.

Then constraint (22) becomes (replacing a_{t-1} by a_t)

$$(29)\quad t_t \leqq w_t \sum_{j=0}^{m} w_{tj}\, g(\bar{r}_j)$$

$$(30)\quad s_t = \ell_t \sum_{j=0}^{m} w_{tj}\, \bar{r}_j$$

$$(31)\quad a_t = \sum_{j=0}^{m} w_{tj}$$

where we have defined $w_{tj} = a_t \omega_{tj}$. Note that the w_{tj} satisfy the AWRs if and only if the ω_{tj} do.

Summarizing, the problem which we seek to solve is <u>Problem AP</u>:

$$\text{maximize} \quad v$$

$$\text{subject to} \quad v \leqq \sum_{t=1}^{T} t_t$$

$$(26)\quad t_t \leqq w_t \sum_{j=0}^{n} z_{tj} f(\bar{v}_j)$$

$$(27)\quad m_t = \sum_{j=0}^{n} z_{tj} \bar{v}_j$$

$$(28)\quad a_t = \sum_{j=0}^{n} z_{tj}$$

$$(29)\quad t_t \leqq w_t \sum_{j=0}^{m} w_{tj} g(\bar{r}_j)$$

$$(30)\quad s_t = \ell_t \sum_{j=0}^{m} w_{tj} \bar{r}_j$$

$$(31)\quad a_t = \sum_{j=0}^{m} w_{tj}$$

$$(14)\quad a_t = k_1 a_{t-1} - x_t - s_t + B_0(1+\alpha_1)^t$$

$$(15)\quad m_t = k_2 a_{t-1} + k_3 m_{t-1} - x_t$$

$$(16) \quad 0 \leqq x_t \leqq k_4 a_{t-1} + k_5 m_{t-1}$$

$$(17) \quad 0 \leqq s_t \leqq k_{6t} a_{t-1}$$

over nonnegative variables v, t_t, z_{tj}, m_t, a_t, s_t, w_{tj}, and x_t for $t=1,\ldots,T$, and $j=1,\ldots,n$ (for the z_{tj}'s) and $j=1,\ldots,m$ (for the w_{tj}'s) .

Also, the sets of variables

$$\{z_{to},\ldots,z_{tn}\}_{t=1}^{T} \quad \text{and} \quad \{w_{t0},\ldots,w_{tm}\}_{t=1}^{T}$$

must satisfy the AWRs.

It is this latter restriction which allows us to classify the above problem AP as a <u>nonconvex</u> problem. An algorithm [4] for a somewhat similar problem was developed at GWU, but assumes that the variables which are required to satisfy the AWRs must sum to 1. Undoubtedly, the methodology of that algorithm could be extended to encompass our problem, but would require some delicate reprogramming of the working code, and, although it would be a very useful addition to our current tools, it would also take several months to complete.

In order to illustrate the solvability of problem AP, we decided to employ LINDO (see [6]), which solves problems with 0-1 variables, and adds constraints to insure that the AWRs are satisfied.

We note that equation (28):

$$a_t = \sum_{j=0}^{n} z_{tj}$$

must be satisfied by a set of variables $\{z_{t0},\ldots,z_{tn}\}$ which satisfy the AWRs. For this set of variables, we introduce 0-1 variables $y_{t1},\ldots,y_{tn}$ and the restrictions

$$(32) \quad 200y_{tj} + z_{t,j-1} + z_{t,j} \geqq a_t \qquad (j=1,\ldots,n)$$

$$(33) \quad \sum_{j=1}^{n} y_{tj} = n-1$$

$$y_{tj} = 0,1$$

Equation (33) guarantees that exactly one variable y_{tj} will be 0, i.e., exactly one of the constraints (32) will be binding. Because of constraint (28), the z_{tj}'s not involved in the binding constraint will be zero.

4. Solution of an Illustrative Example

We solved problem AP with the 0-1 option of LINDO. The following data were used on the example problem.

The parameter values as given in Section 2.

$$(w_1, w_2, w_3, w_4, w_5) = (1,2,1,1,2)$$

$$(\bar{v}_0, \bar{v}_1, \bar{v}_2, \bar{v}_3, \bar{v}_4) = (0, 0.0005, 0.05, 0.1, 1.0)$$

$$(\bar{r}_0, \bar{r}_1, \bar{r}_2, \bar{r}_3, \bar{r}_4) = (0, 0.1, 0.5, 0.9, 1.0)$$

The solution obtained is exhibited in Table 1, and a listing of the program used and all solution values is copied in the appendix. The problem had 102 constraints and 136 variables, 20 of which were 0-1. This is a relatively large problem for LINDO, and took 5529 iterations and 377 branches, involving 10613 pivots.

Table 1

Solution of Example Problem

Time Period	Maintenance Supplied (x_t)	Manpower Supplied (s_t)	Asset Value (a_t)	Maintenance Backlog (m_t)
1	4.86	2.48	100.00	0.05
2	3.91	2.45	101.29	0.14
3	4.05	2.46	102.61	0.13
4	4.07	2.46	104.12	0.16
5	4.17	2.47	105.75	0.00

optimal value = 711.19

APPENDIX

PROBLEM LISTING

and

COMPLETE SOLUTION

Problem Listing

```
MAX        EAV
SUBJECT TO
     2)        X1 +   M1 =      4.9144
     3)        X1 <=    4.98
     4)    - 0.96667 A1 +   A2 +   X2 =      8.4872
     5)    - 0.04 A1 - 0.91444 M1 +   X2 +   M2 =      0
     6)    - 0.04 A1 - 0.98 M1 +   X2 <=    0
     7)    - 0.96667 A2 +   A3 +   X3 =      8.7418
     8)    - 0.04 A2 - 0.91444 M2 +   X3 +   M3 =      0
     9)    - 0.04 A2 - 0.98 M2 +   X3 <=    0
    10)    - 0.96667 A3 +   A4 +   X4 =      9.0041
    11)    - 0.04 A3 - 0.91444 M3 +   X4 +   M4 =      0
    12)    - 0.04 A3 - 0.98 M3 +   X4 <=    0
    13)    - 0.96667 A4 +   A5 +   X5 =      9.2742
    14)    - 0.04 A4 - 0.09144 M4 +   M5 +   X5 =      0
    15)    - 0.04 A4 - 0.98 M4 +   X5 <=    0
    16)      0.0005 Z11 + 0.05 Z12 + 0.1 Z13 -   M1 +   Z14 =      0
    17)      0.0005 Z21 + 0.05 Z22 + 0.1 Z23 -   M2 +   Z24 =      0
    18)      0.0005 Z31 + 0.05 Z32 + 0.1 Z33 -   M3 +   Z34 =      0
    19)      0.0005 Z41 + 0.05 Z42 + 0.1 Z43 -   M4 +   Z44 =      0
    20)      0.0005 Z51 + 0.05 Z52 + 0.1 Z53 -   M5 +   Z54 =      0
    21)       Z10 +   Z11 +   Z12 +   Z13 -   A1 +   Z14 =      0
    22)       Z20 +   Z21 +   Z22 +   Z23 -   A2 +   Z24 =      0
    23)       Z30 +   Z31 +   Z32 +   Z33 -   A3 +   Z34 =      0
    24)       Z40 +   Z41 +   Z42 +   Z43 -   A4 +   Z44 =      0
    25)       Z50 +   Z51 +   Z52 +   Z53 -   A5 +   Z54 =      0
    26)      200 Y11 +   Z10 +   Z11 -   A1 >=    0
    27)      200 Y12 +   Z11 +   Z12 -   A1 >=    0
    28)      200 Y13 +   Z12 +   Z13 -   A1 >=    0
    29)      200 Y14 +   Z13 -   A1 +   Z14 >=    0
    30)       Y11 +   Y12 +   Y13 +   Y14 =      3
    31)      200 Y21 +   Z20 +   Z21 -   A2 >=    0
    32)      200 Y22 +   Z21 +   Z22 -   A2 >=    0
    33)      200 Y23 +   Z22 +   Z23 -   A2 >=    0
    34)      200 Y24 +   Z23 -   A2 +   Z24 >=    0
    35)       Y21 +   Y22 +   Y23 +   Y24 =      3
    36)      200 Y31 +   Z30 +   Z31 -   A3 >=    0
    37)      200 Y32 +   Z31 +   Z32 -   A3 >=    0
    38)      200 Y33 +   Z32 +   Z33 -   A3 >=    0
    39)      200 Y34 +   Z33 -   A3 +   Z34 >=    0
    40)       Y31 +   Y32 +   Y33 +   Y34 =      3
    41)      200 Y41 +   Z40 +   Z41 -   A4 >=    0
    42)      200 Y42 +   Z41 +   Z42 -   A4 >=    0
    43)      200 Y43 +   Z42 +   Z43 -   A4 >=    0
    44)      200 Y44 +   Z43 -   A4 +   Z44 >=    0
    45)       Y41 +   Y42 +   Y43 +   Y44 =      3
    46)      200 Y51 +   Z50 +   Z51 -   A5 >=    0
    47)      200 Y52 +   Z51 +   Z52 -   A5 >=    0
    48)      200 Y53 +   Z52 +   Z53 -   A5 >=    0
    49)      200 Y54 +   Z53 -   A5 +   Z54 >=    0
    50)       Y51 +   Y52 +   Y53 +   Y54 =      3
```

Problem Listing - Continued

```
 51)     EAV -   T1 -   T2 -   T3 -   T4 -   T5 <=    0
 52)     Z10 + 0.99958 Z11 + 0.19908 Z12 + 0.01509 Z13 -  T1 >=    0
 53)    2 Z20 + 1.99916 Z21 + 0.39816 Z22 + 0.03018 Z23 -  T2 >=    0
 54)     Z30 + 0.99958 Z31 + 0.19908 Z32 + 0.01509 Z33 -  T3 >=    0
 55)     Z40 + 0.99958 Z41 + 0.19908 Z42 + 0.01509 Z43 -  T4 >=    0
 56)    2 Z50 + 1.99916 Z51 + 0.39816 Z52 + 0.03018 Z53 -  T5 >=    0
 57)     A1 +   X1 =    104.91
 58)  -  A1 +   W10 +   W11 +   W12 +   W13 +   W14 =     0
 59)  -  A2 +   W20 +   W21 +   W22 +   W23 +   W24 =     0
 60)  -  A3 +   W30 +   W31 +   W32 +   W33 +   W34 =     0
 61)  -  A4 +   W40 +   W41 +   W42 +   W43 +   W44 =     0
 62)  -  A5 +   W50 +   W51 +   W52 +   W53 +   W54 =     0
 63)  -  T1 + 0.00006 W11 + 0.10938 W12 + 0.88574 W13 +   W14 >=    0
 64)  -  T2 + 0.00011 W21 + 0.21875 W22 + 1.77147 W23 + 2 W24 >=    0
 65)  -  T3 + 0.00006 W31 + 0.10938 W32 + 0.88574 W33 +   W34 >=    0
 66)  -  T4 + 0.00006 W41 + 0.10938 W42 + 0.88574 W43 +   W44 >=    0
 67)  -  T5 + 0.00011 W51 + 0.21875 W52 + 1.77147 W53 + 2 W54 >=    0
 68)    0.1 W11 + 0.5 W12 + 0.9 W13 +   W14 - 40.404 S1 =     0
 69)    0.1 W21 + 0.5 W22 + 0.9 W23 +   W24 - 40.81219 S2 =     0
 70)    0.1 W31 + 0.5 W32 + 0.9 W33 +   W34 - 41.2244 S3 =     0
 71)    0.1 W41 + 0.5 W42 + 0.9 W43 +   W44 - 41.6408 S4 =     0
 72)    0.1 W51 + 0.5 W52 + 0.9 W53 +   W54 - 42.0614 S5 =     0
 73)     S1 <=   2.475
 74)  - 0.0245 A1 +   S2 <=    0
 75)  - 0.02426 A2 +   S3 <=    0
 76)  - 0.02401 A3 +   S4 <=    0
 77)  - 0.02377 A4 +   S5 <=    0
 78)  -  A1 +   W10 +   W11 + 200 YW11 >=    0
 79)  -  A1 +   W11 +   W12 + 200 YW12 >=    0
 80)  -  A1 +   W12 +   W13 + 200 YW13 >=    0
 81)  -  A1 +   W13 +   W14 + 200 YW14 >=    0
 82)     YW11 +   YW12 +   YW13 +   YW14 =     3
 83)  -  A2 +   W20 +   W21 + 200 YW21 >=    0
 84)  -  A2 +   W21 +   W22 + 200 YW22 >=    0
 85)  -  A2 +   W22 +   W23 + 200 YW23 >=    0
 86)  -  A2 +   W23 +   W24 + 200 YW24 >=    0
 87)     YW21 +   YW22 +   YW23 +   YW24 =     3
 88)  -  A3 +   W30 +   W31 + 200 YW31 >=    0
 89)  -  A3 +   W31 +   W32 + 200 YW32 >=    0
 90)  -  A3 +   W32 +   W33 + 200 YW33 >=    0
 91)  -  A4 +   W33 +   W34 + 200 YW33 + 200 YW34 >=    0
 92)     YW31 +   YW32 +   YW33 +   YW34 =     3
 93)  -  A4 +   W40 +   W41 + 200 YW41 >=    0
 94)  -  A4 +   W41 +   W42 + 200 YW42 >=    0
 95)  -  A4 +   W42 +   W43 + 200 YW43 >=    0
 96)  -  A4 +   W43 +   W44 + 200 YW44 >=    0
 97)     YW41 +   YW42 +   YW43 +   YW44 =     3
 98)  -  A5 +   W50 +   W51 + 200 YW51 >=    0
 99)  -  A5 +   W51 +   W52 + 200 YW52 >=    0
100)  -  A5 +   W52 +   W53 + 200 YW53 >=    0
101)  -  A5 +   W53 +   W54 + 200 YW54 >=    0
102)     YW51 +   YW52 +   YW53 +   YW54 =     3
END
 NUMBER INTEGER VARIABLES=    20
```

The Complete Solution

OBJECTIVE FUNCTION VALUE

1) 711.193726

VARIABLE	VALUE	REDUCED COST
Y11	0.000000	-426.513763
Y12	1.000000	0.000000
Y13	1.000000	0.000000
Y14	1.000000	0.000000
Y21	1.000000	0.000000
Y22	0.000000	-137.946289
Y23	1.000000	0.000000
Y24	1.000000	0.000000
Y31	1.000000	0.000000
Y32	0.000000	-204.508804
Y33	1.000000	0.000000
Y34	1.000000	0.000000
Y41	1.000000	0.000000
Y42	0.000000	-384.489441
Y43	1.000000	0.000000
Y44	1.000000	0.000000
Y51	0.000000	0.000000
Y52	1.000000	0.000000
Y53	1.000000	0.000000
Y54	1.000000	0.000000
YW11	1.000000	-28.529995
YW12	1.000000	-28.529995
YW13	1.000000	-28.529995
YW14	0.000000	0.000000
YW21	1.000000	-55.762081
YW22	1.000000	-55.762081
YW23	1.000000	-55.762081
YW24	0.000000	0.000000
YW31	1.000000	0.000000
YW32	1.000000	0.000000
YW33	1.000000	0.000000
YW34	0.000000	0.000000
YW41	1.000000	-24.913073
YW42	1.000000	-24.913073
YW43	1.000000	-24.913073
YW44	0.000000	0.000000
YW51	1.000000	-57.059765
YW52	1.000000	-57.059765
YW53	1.000000	-57.059765
YW54	0.000000	0.000000
EAV	711.193726	0.000000

The Complete Solution - Continued

Z10	0.000000	0.001067
Z11	100.045631	0.000000
Z12	0.000000	2.026954
Z13	0.000000	1.920272
Z20	0.000000	0.690080
Z21	99.568626	0.000000
Z22	1.717557	0.000000
Z23	0.000000	0.661322
Z30	0.000000	1.023061
Z31	101.011032	0.000000
Z32	1.595954	0.000000
Z33	0.000000	0.980426
Z40	0.000000	1.923419
Z41	102.013145	0.000000
Z42	2.111281	0.000000
Z43	0.000000	1.843263
Z50	105.748558	0.000000
Z51	0.000000	0.000143
Z52	0.000000	0.014265
Z53	0.000000	0.028530
A1	100.045631	0.000000
X1	4.864372	0.000000
M1	0.050026	0.000000
A2	101.286194	0.000000
X2	3.912110	0.000000
M2	0.135460	0.000000
A3	102.607109	0.000000
X3	4.045224	0.000000
M3	0.130316	0.000000
A4	104.124428	0.000000
X4	4.066876	0.000000
M4	0.156571	0.000000
A5	105.748550	0.000000
M5	0.000001	0.000000
X5	4.179297	0.000000
Z14	0.000000	0.000000
Z24	0.000000	0.000000
Z34	0.000006	0.000000
Z44	0.000000	0.000000
Z54	0.000000	0.285299
T1	99.993393	0.000000
T2	199.737488	0.000000
T3	101.286430	0.000000
T4	102.390617	0.000000
T5	207.785812	0.000000
W10	0.000000	0.000000
W11	0.000000	0.114210
W12	0.000000	0.461930
W13	0.457153	0.000000
W14	99.386478	0.000000

The Complete Solution - Concluded

W20	0.000000	0.000000
W21	0.000000	0.223225
W22	0.000000	0.902888
W23	12.404925	0.000000
W24	86.881271	0.000000
W30	1.320671	0.000000
W31	0.000000	0.093206
W32	0.000000	0.364284
W33	0.000000	0.013303
W34	101.286430	0.000000
W40	0.000000	0.000000
W41	0.000000	0.099731
W42	0.000000	0.403383
W43	15.173573	0.000000
W44	88.950859	0.000000
W50	0.000000	0.000000
W51	0.000000	0.228420
W52	0.000000	0.923899
W53	16.239845	0.000000
W54	89.508705	0.000000
S1	2.475000	0.000000
S2	2.451368	0.000000
S3	2.456957	0.000000
S4	2.464099	0.000000
S5	2.475537	0.000000

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